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Spin Orbit Effects and Superconductivity in Oxide Materials

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Abstract

In a variety of materials superconductivity is associated with the existence of a quantum critical point (QCP). In the case of the hole doped cuprates there is evidence which suggests that the important quantum degrees of freedom near the superconducting critical point are localized charge and spin density fluctuations. We argue that if these degrees of freedom are strongly coupled by spin-orbit interactions, a new type of quantum criticality arises with monopole-like quasi-particles as the important quantum degrees of freedom,. In layered material this type of quantum criticality can be modeled using a 2-dimensional non-linear Schrodinger equation with an SU(N) gauge field. We exhibit a pairing wave function for quasi-particles that has topological order and anisotropic properties. The superconducting transition would in some respects resemble a KT transition.

1. Introduction

In a variety of materials it has been observed that superconductivity is associated with quantum critical magnetic fluctuations [1,2,3] Such an association also appears in the case of the electron doped high T_c superconducting cuprates. However, in the case of the hole doped cuprates the association of superconductivity with a quantum critical point (QCP) is less obvious, although anomalous behavior of transport properties for temperatures above T_c may be evidence for proximity to a QCP [4]. These measurements suggest that the QCP is hidden inside the superconducting region of the phase diagram.

In the case of the occurrence of superconductivity in the pyrochlore $\text{Ce}_2\text{Re}_2\text{O}_7$, there is a strong hint as to the essential physics involved by virtue of the circumstance that the superconductivity at low temperature appears to be related to a “ferroelectric” transition associated with a cubic to a tetragonal martensitic transformation that occurs near 200 K [5]. The first order structural phase transition appears to be accompanied by a continuous second order phase transition that fits Anderson and Blount’s description [6] of a “ferroelectric” transition in a conductor. Although it is not possible to have macroscopic electric fields in a metal because of screening, short range electric field fluctuations can occur, and in a “bad metal” these electric field fluctuations can extend over mesoscopic distances. In the hole doped cuprates there is abundant evidence that unscreened electric fields occur along the c-axis [7]. It is our thesis that it is spin-orbit interactions between charge carriers and electric fields that is responsible for the superconductivity of $\text{Ce}_2\text{Re}_2\text{O}_7$ and the hole doped cuprates.

In the case of the hole doped cuprates, evidence that charge and spin fluctuations might be associated with the occurrence of superconducting is provided by evidence for a spin glass phase in LaCuO_4 doped with Sr. The existence of a spin glass phase for hole dopings between the antiferromagnetic and superconducting regions was suggested some time ago by magnetization measurements [8]. More recently Panagopolous [9] has used muon spin rotation to show that a spin glass phase in LaCuO_4 doped with Sr actually extends all the way from zero hole doping to a hole doping near to optimal doping; i.e. roughly in the middle of the superconducting region. Thus it appears quite plausible that there is an intimate connection between the critical fluctuations associated with this spin glass phase and high T_c superconductivity.

Although to our knowledge it has never been suggested that spin orbit interactions play an important role in the cuprates, it has been known for a long time that spin orbit effects lead to interesting effects in transition metal oxides. For example, spin orbit effects are responsible for the magnetic anisotropy of ferrite materials. In Fe_2O_4 doped with Co spin orbit interactions in the presence of a Jahn-Teller distortion lead to a switching of the easy axis from $\langle 100 \rangle$ to $\langle 111 \rangle$. In the case of the cuprates there may also be interesting effects related to Jahn-Teller distortions; indeed, this was one of the original motivations for looking for superconductivity in the cuprates [10]. However, the more immediate reason for our interest in spin orbit effects in oxide materials is the experimental discovery of the spin Hall effect in semiconductors.

2. Spin currents in layered conductors

Murakami, Nagaosa, and Zhang [11] and Sinova, et. al. [12] have pointed out that in the presence of an external electric field the Rashba spin orbit interaction term in a semiconductor will give rise to pure spin currents; i.e. a spin polarization current without a flow of electric charge. The predicted form of the spin current suggests that if an electric field is applied in the plane of a semiconductor channel with a large Rashba coupling, then equal and opposite out of plane spin polarizations will appear at the edges of the channel. In contrast with the anomalous Hall effect these edge spin polarizations are not accompanied by a buildup of electric charge. Experimental evidence for the existence of such an intrinsic spin Hall effect has been found in the case of a 2-dimensional hole liquid in a semiconductor heterostructure [13]. In the presence of impurities there is in addition to the intrinsic effect an extrinsic spin Hall effect associated with the impurities. In the following we will explore whether a spin Hall effect associated with unscreened electric fields, e. g. along the c-axis, might play a role in high T_c superconductivity.

The spin current inside a 2-dimensional conductor is defined to be

$$j_i^{\uparrow} = \frac{1}{2} \int d^2k \langle v_i^{\uparrow} + v_i^{\downarrow} \rangle, \quad (1)$$

where the integral extends over the occupied states of the conduction band. In general, quantum ground states can have non-vanishing spin currents, but only if inversion symmetry is broken and spin-orbit effects are important. In the presence of a uniform electric field E the induced spin current due to spin orbit interactions will have the form [11]:

$$j_i^{\uparrow} = \sigma_s \epsilon_{ij} E_j, \quad (2)$$

where σ_s is the spin Hall “conductivity”. In a metal or semiconductor σ_s will be proportional to the Fermi momentum, and in a typical semiconductor will actually be comparable in magnitude to the ordinary electrical conductance. Of course, inside a good conductor there will be no macroscopic electric field due to screening, and so the spin current induced by the external field will vanish. However, in a conductor with a low carrier density there can be local electric fields and hence spin currents due to charge fluctuations even in the absence of an external field. Furthermore, and this is the point of greatest interest to us, due to spin-orbit interactions the local charge fluctuations and induced spin currents might be viewed as collective excitations of an electronic quantum fluid.

In order to describe the effect of screening on spin-orbit induced effects, we introduce an equation which relates the Lorentz magnetic field seen by conduction electrons to an effective screening length. In a 2-dimensional conductor and in the presence of screening Gauss’ law becomes [14]

$$\nabla \cdot E - \epsilon F_{ij} = 0, \quad (3)$$

where B_{eff} is the out of plane component (i.e. perpendicular to the layer) of the Lorentz magnetic field seen by conduction electrons, ϵ is the charge per unit area, and ϵ is the inverse of a effective screening length for spin orbit effects.. Averaging Eq. (2) over a mesoscopic area yields

$$B_{\text{eff}} = \frac{1}{\epsilon} \epsilon, \quad (4)$$

In a layered conductor with a low carrier density Eq.s (2) and (4) might be interpreted as constitutive equations for a quantum fluid, somewhat analogous to the infinite conductivity MHD equations.

In order to obtain a picture of the collective excitations in the 2-dimensional quantum fluid defined by Eq’s. (2) and (4), we introduce a non-linear Schrodinger equation for spin polarized carriers

$$i\hbar \frac{\partial \psi}{\partial t} = \psi \frac{1}{2m} D^2 \psi + e A_0 \psi \mp g |\psi|^2 \psi, \quad (5)$$

where \mp refers to spin up or down along the z-axis, i.e. perpendicular to the conducting plane, and $D_i = \partial_i - ieA_i$. The gauge fields A_0 and A_i do not satisfy Maxwell's equations, but instead are determined from the charge and current densities by solving Eq's (2) and (3). As shown by Jackiw and Pi [14] the time independent version of Eq. (5) can be exactly solved if one assumes Eq. (4) holds and that $g = \pm e^2/m$. In this case there are vortex-like solutions with associated spin currents that satisfy Eq. (2) with $E_i = -\partial_i A_0$, $A_0 = \phi/(2\ell)$, and $\psi_s = \psi$. In the case of a single vortex, the spin current resembles the flow around a quantized vortex core in superfluid helium.

In a 3-dimensional layered material one may also have to take into account interlayer tunneling. Indeed as originally suggested by Anderson [15] interlayer tunneling in the hole doped cuprates may play an important role in high temperature superconductivity. However, rather than following Anderson's discussion, we follow our earlier scheme [16,17] for dealing with the effects of interlayer tunneling by introducing a non-linear Schrodinger equation for an SU(N) wave function ψ for spin polarized carriers, where N is the number of layers:

$$i\hbar \frac{\partial \psi}{\partial t} = \psi \frac{1}{2m} D^2 \psi + e[A_0, \psi] + g[[\psi^\dagger, \psi], \psi], \quad (6)$$

where A_0 and A_i are now SU(N) gauge fields and $D = \partial + ie[A, \cdot]$. Following Grossman [18] we choose A_i to lie in the Cartan subalgebra of SU(N) and ψ to be a ladder operator within the adjoint representation.

If we choose the A_i to lie within the Cartan subalgebra of SU(N), the out of plane Lorentz magnetic field $B_{\text{eff}} = \partial_x A_y - \partial_y A_x + [A_x, A_y]$ will be given by

$$B_{\text{eff}} = -\frac{e}{\ell} [\psi^\dagger, \psi]. \quad (7)$$

Instead of Eq. (4), which relates the effective magnetic field to the charge density per unit area in a single layer, we now have an equation which allows the effective magnetic field to vary from layer to layer, but for each layer the effective field is still proportional to the spin polarized charge density per unit area. As in the 2-dimensional case Eq's. (6) and (7) can be exactly solved when $g = \pm e^2/m$.

The in plane electric field $E_j = -\partial_i A_0 - [A_i, A_0]$ will be given by

$$E_j = \frac{e}{m\ell} \ell_j j_j, \quad (8)$$

where j_j is the current

$$j_i = \frac{i}{2} (\partial_i D_i + D_i \partial_i) . \quad (9)$$

Thus the in plane electric field and spin polarization current are formally related in much the same way as the electric field and spin current in a 2-dimensional semiconductor with a large spin orbit coupling. Of course, the spin polarized current (9) also carries electric charge and so it will only be possible to identify Eq. (8) with a pure spin current when we consider pairing of self-dual and anti-self-dual solitons.

In the limit $N \rightarrow \infty$ the effective magnetic field (7) becomes [16]

$$B_j = \frac{e}{\hbar} \sum_k \partial_k |X_j - X_k| , \quad (10)$$

where X is now a 3-dimensional coordinate. Thus the vortex-like solitons solutions for a single layer morph into dyon-like quasi-particles in the limit $N \rightarrow \infty$. These dyon-like quasi-particles resemble polarons in that they carry electric charge, but differ from polarons in that they also carry an effective magnetic charge and are associated with spin currents.

3. Superconducting wave function

The ground state wave function for the non-linear Schrodinger equation (6) assuming that Eq.(7) also holds and $g = \pm e^2/\hbar$ has the form [16]

$$\Psi = f(\bar{z}) \prod_{k>j} \left(\frac{R_{jk} + U_{jk}}{R_{jk} - U_{jk}} \right)^{1/2} , \quad (11)$$

where $R_{jk}^2 = U_{jk}^2 + 4 (z_j - z_k)(\bar{z}_j - \bar{z}_k)$, $U_{jk} = U_j - U_k$, and $f(\bar{z})$ is an entire function of the \bar{z}_j .

The product on the right hand side of (1) replaces the $\prod_{k>j} |z_j - z_k|^{\hbar^2/4}$ factor in Laughlin's fractional quantum Hall effect wave function [19]. Like the Laughlin quasi-particles which obey fractional statistics, the dyon-like quasi-particles (referred to as "chirons" in ref. 16) described by the wave function (1) have interesting holonomy properties. The complex phase or "action" associated with a quasi-particle whose 3-dimensional position is parameterized by $X \equiv (z, U)$ where $z = x + iy$ will be given by

$$\Phi = \frac{1}{2} \sum_j \ln \frac{R_j + U - U_j}{R_j - U + U_j} , \quad (12)$$

where $R_j^2 = (U - U_j)^2 + 4(z - z_j)(\bar{z} - \bar{z}_j)$. The change in phase moving around a quasi-particle located at $z = z_k$, $U = U_k$ is

$$\oint_k \gamma dz = \frac{1}{2} \oint_i \oint_k \frac{U_{ji}}{R_i} \frac{dz}{z - z_i} = i\gamma. \quad (13)$$

Thus for cyclic changes in z our 3-dimensional quasi-particles behave like anyons with $1/2$ fractional statistics. It should be noted though that the holonomy is non-trivial only if $U_{jk} \neq 0$, so that in our case the effects of the non-trivial holonomy necessarily involve 3 spatial dimensions. It can be shown [16] that our model wave function (11) describes a topologically non-trivial self-dual or anti-self-dual manifold in 4-dimensions. Furthermore pairing of self-dual and an anti-self-dual solutions describes a topologically trivial manifold in 4-dimensions [17]. The next question is whether this paired wave function describes a superfluid?

Actually the quasi-particle phase (12) suggests a connection with the Kosterlitz-Thouless (KT) transition in helium films [20]. A KT-like transition also occurs in superconducting films and the XY model for a 2-dimensional ferromagnet. The KT transition in the XY model consists of a condensation of vortex and anti-vortex configurations of planar spins into bound state pairs. Each XY spin is described by an angle θ ; and the vortex configurations implicated in the KT transition have the form:

$$\theta(z) = m_i \text{Im} \ln(z - z_i), \quad (14)$$

where the integer m_i is the quantized circulation of the vortex (or anti-vortex if m_i is negative) located at z_i . Expression (14) is remarkably similar to our expression (12) for the phase of our quasi-particles. On the high temperature side of the KT transition the XY model is equivalent to a 2-dimensional gas of vortices with Coulomb-like interactions, while on the low temperature side of the transition the model becomes a scale invariant theory of massless spin waves. It seems plausible to conjecture that the pairing of self-dual and anti-self-dual quasi-particles should resemble the conformally invariant KT phase, and therefore the paired state may also possess similar properties.

The conformally invariant phase of the XY model possesses many features in common with conventional superconductors with long range off-diagonal order (ODLRO) such as a superfluid ground state and persistent currents. Therefore on the basis of the similarity of our 3-dimensional quasi-particles and KT vortices we expect that pairing of self-dual and anti-self-dual quasi-particles does indeed describe a superfluid state – though perhaps a state with only topological order rather than ODLRO. Amusingly observations of a Nernst effect in the underdoped cuprates suggest that the superconducting transition in the hole doped cuprates is actually a KT-like transition [21]. It is tempting to speculate that this KT-like

behavior is just a reflection of the fact that our 3-dimensional quasi-particles retain many of the characteristics of the vortex-like soliton solutions of the 2-dimensional spin Hall effect and equations (4) and (5).

4. Conclusion

Although one doesn't normally think of spin orbit interactions as being important in cuprate materials, the discovery of the spin Hall effect in semiconductors raises the question as to whether similar kinds of effects might not exist in transition metal oxides which are also poor conductors. Indeed applying the same constitutive equations to a layered conductor as have been used to describe the spin Hall effect, we have arrived at a theory of quantum critical behavior where the quantum degrees of freedom are dyon-like quasi-particles. We argued that attraction between these quasi-particles leads to a superfluid state with some similarities to the KT state. In a sense the mysterious "glue" responsible for high T_c superconducting is just the attraction between magnetic monopoles of opposite charge.

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